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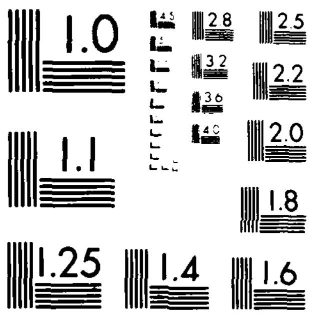

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THE CHANGING SCENE IN COMPUTATIONAL FLUID DYNAMICS

AFOSR-TR- 84-0115

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The evolution of numerical techniques for solving problems in Fluid Dynamics is followed, in outline, from the days when Digital Computers were first available, at the end of the Second World War, to the present time, when the Computer Aerodynamic Simulator is being assembled. In this period the range of numerical methods has been broadened five fold, while the speed and capacity of computers have increased by several orders of magnitude. Two areas close to the author's interests are selected to illustrate these changes. The first concerns the extension of the Method of Integral Relations to apply to laminar and turbulent boundary layer problems, including internal flows, separated flows and turbulent mixing flows. The second area deals with unsteady inviscid compressible flow in one or more dimensions and a discussion is given of the relative merits of Godunov and Glimm techniques.

INTRODUCTION

The need for numerical, as distinct from analytical, methods to solve problems in Fluid Dynamics first arose when problems in Gas Dynamics presented themselves. These are connected with combustion and explosive processes and are essentially non-linear. The equations of motion governing such problems are hyperbolic and, in principle, could be solved numerically by the Method of Characteristics. This has a long history going back to the pioneer paper by Massau (1900). The foundations of the method are laid out in Courant, Friedrichs and Lewy (1928) and developed for application in Courant and Friedrichs (1978).

Two difficulties arise in applying the Method of Characteristics to Gas Dynamic problems. Firstly, shock waves appear frequently in such problems and need to be fitted on an ad hoc basis into the characteristic network. Secondly, this network does not have uniform mesh spacing, is non-orthogonal, curvilinear and frequently highly skewed.

One of the earliest methods avoiding these drawbacks was the Finite Difference method of von Neumann and Richtmyer (1950). This applied to unsteady spherical flow in Lagrangian coordinates, using finite differences in the original orthogonal independent variables, and treated shocks by the introduction of artificial viscosity. This was refined later in the well-known Lax-Wendroff(1964) method. The development of these methods is well described in Richtmyer and Morton (1972).

The search for better numerical techniques to solve Gas Dynamic problems was given further impetus in the Sputnik era, with the need to solve the flow field problem for a space vehicle on re-entry into the earth's atmosphere, the so-called Blunt Body problem.

This led to a spate of new techniques, mostly developed in the USSR. Firstly, Godunov's method (1959) was presented as a novel way to solve the Lagrangian equation in unsteady flow but grew into one for solving the unsteady Eulerian equations in two and three dimensions —yielding the solution to the blunt body problem as the steady flow, asymptotic, limit of the unsteady solution. Godunov used

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discontinuity breakdown formulae, in place of finite difference formulae, a significant departure from the previous approaches of Lax-Wendroff and others.

A solution to the blunt body problem predating that of Godunov was obtained by Belotserkovskii (1958) who solved the equations of steady motion for symmetrical flow past a circular cylinder or sphere, of mixed elliptic-hyperbolic type, by the first formulation of the Method of Integral Relations due to Dorodnitsyn (1956). In this method moments of the equations of motion are integrated across the flow field, the integrands are represented by suitable interpolation functions and the problem is reduced to the integration of ordinary differential equations in the coefficients of these functions.

The steady flow version of the Blunt Body problem was also solved by Telenin's method (Gilinskii, Telenin and Tinyakov, 1964), in which the unknown is represented by an interpolation function in one of the coordinates, in the original equations of motion. The Method of Lines (Jones and South, 1979) is a similar technique applied over three or five mesh points instead of over the whole flow field.

The last method developed for high speed steady flow problems is due to Babenko and others. The version applied to purely supersonic flow is by Babenko, Voskresenskii, Lyubimov and Rusanov (1964) while the blunt body version was developed by Lyubimov and Rusanov (1970). This is a sophisticated finite difference method applicable to any steady high speed inviscid flow problem, especially in three dimensions.

When viscous effects are important we need to solve the Navier-Stokes equations, either in their original, or approximate, boundary layer form. The full equations are non-linear and elliptic. Several Finite Difference methods have been developed for these but they become prohibitively expensive as the governing Reynolds number is increased. In the low speed range Finite Element methods have been proposed for flow fields limited by boundaries (cavities and steps) while Spectral methods have been used for investigation of flow structures in unlimited regions. A full account of these is given in Peyret-Taylor (1983).

In many applications viscous effects only need to be considered near boundaries and it is sufficient to solve the boundary layer equations in these regions in interaction with inviscid flow in the outer regions. Many finite difference methods have been developed for the boundary layer equations. At the present time, however, these can be replaced by more recently developed techniques such as the Method of Integral Relations, Finite Element methods, Galerkin techniques and Spectral Methods. These are described in full detail by Fletcher (1983).

The remainder of this article deals with two topics connected with the author's own research, Recent Applications of the Method of Integral Relations to Turbulent and Internal Flows and Recent Developments in Methods for Problems in Gas Dynamics and Propagation of Large Amplitude Surface Waves.

RECENT APPLICATIONS OF THE METHOD OF INTEGRAL RELATIONS

The Method of Integral Relations was first formulated for Viscous Incompressible Boundary Layer Problems by Dorodnitsyn (1960). This formulation was extended to compressible flows by Pavlovskii (1963) and to flows with wall injection by Liu (1962). The early applications were all to attached flows but extensions of the method to apply to separated and reversed flows were made by Nielsen, Goodwin and Kuhn (1969) and by Holt (1966,1967) and Holt and Lu (1975).

In the original formulation the basic integral relation is derived by factoring the continuity equation by one of a set of weighting functions $f(u)$ and the streamwise momentum equation by its derivative $f'(u)$ where u is the streamwise velocity component. The results are added and integrated across the boundary layer, using u, rather than η (the transverse coordinate), as variable of integration. The functions $f(u)$ belong to a complete set and taken in fact as integral powers of $(1-u)$ [u is made dimensionless with respect to velocity just outside the boundary layer]. The integrands in the integral relation contain the unknown transverse velocity gradient $\partial u/\partial\eta$, which, in the Dorodnitsyn formulation, is represented as a polynomial in u, factored by a term (1-u) to ensure approach to zero at the boundary layer edge. Successive integral relations in the nth approximation yield n ordinary differential equations for the coefficients in the $(\partial u/\partial\eta)$ polynomial. The matrix of these equations (for the first derivatives) is dense and the system becomes progressively more ill-conditioned as the order of approximation is increased.

To eliminate this defect Fletcher and Holt (1975) proposed a modified formulation of the Method of Integral Relations. In this, the weighting functions f(u) belong to a complete orthonormal set in (0,1). Moreover, the unknown $(\partial u/\partial\eta)$ is expanded in combinations of the same orthonormal set, factored by (1-u). The resulting ordinary differential equations for the coefficients in the expansion then have a sparse matrix with diagonal elements only. The integrals which occur can all be evaluated by quadratures and the modified version can be applied to attached flows at any level of approximation.

The extension of the Dorodnitsyn MIR to separated flows introduces a great deal of tedious algebra, even in the lowest order approximation and it would be useful to extend the Fletcher-Holt modified version for such flows. The difficulty here is that a factor $(u+\alpha)^{\frac{1}{2}}$ must be included in the representation of $(\partial u/\partial\eta)$ to take account of reversed flow between $u=0$ (at the wall) and $u=-\alpha$, with a vertical tangent in the $\partial u/\partial\eta - u$ curve at $u=-\alpha$.

A possible orthonormal formulation for separated flows is now given, developed as a course term project at the University of California, Berkeley by O. Ozcan and R.-J. Yang. This uses Chebychev polynomials. This is followed by an orthonormal formulation of MIR for turbulent boundary layers, applied to model turbulent flows by Yeung and Yang (1981) and to the turbulent wall jet problem by Yang and Holt (1983).

ORTHONORMAL FORMULATION OF MIR FOR SEPARATING FLOWS

When the original Method of Integral Relations is applied to the incompressible laminar boundary layer equations in Dorodnitsyn variables, the following basic integral relation results

$$\frac{d}{d\xi}\int_A^B u f_k \theta du = \frac{\dot{U}_1}{U_1}\int_A^B (1-u^2) f_k'(u)\theta du + \left.\frac{f_k'(u)}{\theta}\right|_A^B - \int_A^B \frac{f_k''}{\theta} du$$
$$+ \left. u f_k\right|_B \frac{d\eta_2}{d\xi} - \left. u f_k\right|_A \frac{d\eta_1}{d\xi} - \left. w f_k(u)\right|_A^B , \qquad (1)$$

where ξ is the streamwise coordinate, u the velocity component in the ξ direction, θ the reciprocal of $(\partial u/\partial\eta)$, η the transverse coordinate and $f_k(u)$ are weighting functions. The points A and B correspond to the u limits of integration over the section of boundary layer in question. The values η_1,η_2 correspond to η at A and B, respectively.

The velocity field is divided into two parts in the attached (but retarded) boundary layer region and into three parts in the separated region.

Attached flow

In the velocity range $0 < u < \varepsilon$, where ε is small $(\varepsilon \sim 0.1)$, θ is represented by the second order approximate expression

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$$\theta = \frac{c_o}{(u+\alpha)^{\frac{1}{2}}(1-u)} \tag{2}$$

The following two weighting functions are chosen to simplify the calculation of the integrals in Eq. (1)

$$f_k(u) = (\varepsilon-u)(1-u)(u+\alpha)^{k+\frac{1}{2}} \quad , \quad k = 1,2 \quad . \tag{3}$$

Equations (2) and (3) are substituted in Eq. (1) to yield two ordinary differential equations for c_o and α

$$\frac{d}{d\xi}\left[c_o E_k(\alpha)\right] = F_k(c_o,\alpha) \quad , \quad k = 1,2 \quad , \tag{4}$$

where E_k are polynomials in α, and F_k are polynomials in α, linear in c_o, with coefficients containing ε.

In the velocity range $\varepsilon < u < 1$, θ is represented by the following expression

$$\theta = \frac{1}{(u+\alpha)^{\frac{1}{2}}(1-u)} \left[b_o + \sum_{i=2}^{N} \sum_{j=1}^{N-1} \frac{b_i\sqrt{u+\alpha}}{u(1-u)^{3/4}} \frac{g_i + g_j}{\sqrt{u-\varepsilon}}\right] \tag{5}$$

together with weighting functions

$$f_k = (1-u)^{5/4} g_k \quad , \tag{6}$$

where g_k are Chebychev polynomials, i and k are even, j is odd.

The orthogonality condition for these polynomials is

$$\int_{-1}^{+1} \frac{g_i(U)g_k(U)}{(1-U^2)^{\frac{1}{2}}} dU = \begin{matrix} \pi/2 \text{ when } i = k \\ 0 \text{ when } i \neq k \end{matrix} \tag{7}$$

The limits of integration $\varepsilon \to 1$ are changed to $-1 \to 1$ by means of the change in variable

$$U = \frac{1}{\varepsilon-1}\left[\varepsilon + 1 - 2u\right] \quad . \tag{8}$$

We can show that

$$\underset{\substack{u \to \varepsilon \\ U \to -1}}{\mathrm{Lt}} \frac{g_i + g_j}{(u-\varepsilon)^{\frac{1}{2}}} = 0 \quad .$$

Hence, matching expressions (2) and (5) at $u = \varepsilon$,

$$a_o = b_o$$

Substitution of Eqs. (5) and (6) in Eq. (1) yields

$$\frac{\pi}{2}(N-4)\frac{db_k}{d\xi} = C'(k) \quad , \quad k = 1,\ldots,N \quad , \tag{9}$$

where $C'(k)$ is a linear combination of the unknowns b_n with coefficients which can be evaluated numerically.

Separated flow

In the separated flow region three representations of $\partial u/\partial\eta$ in terms of u are used. In the reversed flow range $-\alpha < u < 0$, close to the wall, we use

$$\theta = -\frac{a_0}{(u+\alpha)^{\frac{1}{2}}(1+\alpha)}$$

with weighting function (10)

$$f = (u+\alpha)^{\frac{3}{2}} .$$

In the intermediate range $-\alpha < u < \varepsilon$, where ε is small and positive, we use

$$\theta = -\frac{a_0}{(u+\alpha)^{\frac{1}{2}}(1-u)}$$

$$f = (\varepsilon-u)(1-u)(u+\alpha)^{\frac{3}{2}} \tag{11}$$

Substitution of Eqs. (10) and (11) into Eq. (1) yields two first order ordinary differential equations for a_0 and α.

In the outer range $\varepsilon < u < 1$ we again use representation (5) for θ with weighting functions (6). The coefficients b_n are determined from the same equation (9).

This approach is currently being applied to model separated flow problems.

APPLICATION OF MIR TO TURBULENT BOUNDARY LAYERS

The key to the extension of the Method of Integral Relations to apply to turbulent boundary layer flow is in the representation of the eddy viscosity as a function of mean streamwise velocity component. This was investigated by Abbott and Deiwert (1968) and by Murphy and Rose (1968). Although their models of eddy viscosity were reasonable these were not well adapted to the original formulation of MIR and produced disappointing results. On the other hand, when the modified, orthonormal, version of MIR is applied to turbulent boundary layers, with representations of eddy viscosity similar to those used previously, comparisons of applications to experimental results in model cases prove to be very satisfactory. This advance is described in Yeung and Yang (1981).

The turblent boundary layer equations in two dimensions may be written

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[(\nu + \frac{\varepsilon}{\rho})\frac{\partial u}{\partial y}\right] , \tag{12}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 , \tag{13}$$

where u and v are the x and y components of mean velocity, p is the mean pressure, ρ the density, ν the kinematic viscosity. The eddy viscosity ε is given by

$$-\rho\,\overline{u'v'} = \varepsilon\frac{\partial u}{\partial y} \tag{14}$$

representing the Reynolds stress.

We introduce the dimensionless variables

$$U = \frac{u}{u_e} , \; V = \frac{vRe^{1/2}}{u_e} , \; \bar{x} = \frac{x}{L} , \; \bar{y} = \frac{yRe^{1/2}}{L} , \; U_e = \frac{u_e}{u_\infty} , \tag{15}$$

where

$$Re = u_\infty L/\nu \quad . \tag{16}$$

Bernoulli's equation gives

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = u_e \frac{du_e}{dx} \tag{17}$$

and Eqs. (12) and (13), in the dimensionless variables, become

$$U\frac{\partial U}{\partial \bar{x}} + V\frac{\partial U}{\partial \bar{y}} = \frac{1}{U_e}\frac{dU_e}{d\bar{x}}(1-U^2) + \frac{1}{U_e}\frac{\partial}{\partial \bar{y}}\left[(1+\frac{\varepsilon}{\mu})\frac{\partial U}{\partial \bar{y}}\right] \quad , \tag{18}$$

$$\frac{\partial U}{\partial \bar{x}} + \frac{\partial V}{\partial \bar{y}} = -\frac{U}{U_e}\frac{dU_e}{d\bar{x}} \quad . \tag{19}$$

To apply the Method of Integral Relations we introduce a complete set of orthonormal functions (over (0,1)) $f_i(U)$, multiply Eq. (18) by f_i', add the result and integrate with respect to $\bar{y}$ across the boundary layer. We then change the variable of integration from $\bar{y}$ to U, introducing the reciprocal of the transverse mean velocity gradient

$$Z = \left(\frac{\partial U}{\partial \bar{y}}\right)^{-1} \quad . \tag{20}$$

The basic integral relation is then

$$\frac{\partial}{\partial \bar{x}}\int_0^1 f_i UZdU = \frac{1}{U_e}\frac{dU_e}{d\bar{x}}\int_0^1[(1-U^2)f_i' - Uf_i]ZdU$$
$$- \frac{1}{U_e} f_i'(0)\frac{1}{Z_o} - \frac{1}{U_e}\int_0^1(1+\frac{\varepsilon}{\mu})\frac{f_i''}{Z}dU \quad . \tag{21}$$

The orthonormal functions $f_i(U)$ are given by

$$f_i(U) = \sum_{k=1}^{i} c_{ik}(1-U)^k \quad , \tag{22}$$

and

$$\int_0^1 f_k f_j \frac{U}{1-U}dU = \delta_{kj} \quad , \tag{23}$$

where δ_{kj} is the Kronecker delta. We represent Z by the factored orthonormal expansion

$$Z = \frac{b_o + \sum_{j=1}^{N-1} b_j f_j(U)}{1-U} \tag{24}$$

The basic integral relation (21) then yields the following set of ordinary differential equations for the unknown coefficients $b_o, b_1, b_2, \ldots$ in Eq. (24):

$$\frac{db_o}{d\bar{x}}\int_0^1 \frac{f_i U}{1-U}dU + \frac{db_i}{d\bar{x}} = \frac{1}{U_e}\frac{dU_e}{d\bar{x}}\int_0^1[(1-U^2)f_i' - Uf_i]ZdU$$
$$- \frac{1}{U_e}\frac{f_i'(0)}{Z_o} - \frac{1}{U_e}\int_0^1(1+\frac{\varepsilon}{\mu})\frac{f_i''}{Z}dU \quad , \quad i = 1,2,\ldots,N-1 \tag{25a}$$

and

$$\frac{db_o}{d\bar{x}}\int_0^1 \frac{f_N U}{1-U}dU = \frac{1}{U_e}\frac{dU_e}{d\bar{x}}\int_0^1[(1-U^2)f_N' - Uf_N]ZdU$$
$$- \frac{1}{U_e} f_N'(0)\frac{1}{Z_o} - \frac{1}{U_e}\int_0^1(1+\frac{\varepsilon}{\mu})\frac{1}{Z}f_N''dU \tag{25b}$$

These can be integrated subject to suitable initial conditions.

The advantages of the modified version of MIR over the original version are now apparent. Firstly, the system of ordinary differential equations [(25a) and (25b)] can easily be reduced to diagonal form while the corresponding equations in the original formulation are highly coupled. Secondly, the representation (24) for Z allows for greater flexibility in reproducing the highly inflected velocity profile characterizing turbulent (as opposed to laminar) boundary layers. Thirdly, in the orthonormal version, the integral

$$\int_0^1 (1 + \frac{\varepsilon}{\mu}) \frac{1}{Z} f_i''(U)dU$$

can be evaluated by quadrature, while in the original version this must be reduced to an algebraic expression. This is a tedious task, increasingly difficult to carry out as the order of approximation is increased, since the term ε/μ is represented by a complicated combination of exponentials and powers in U.

Turbulence modelling

The eddy viscosity ε/μ is represented by two formulae, one based on a model due to Spalding (1961) and Kleinstein (1967) applicable near the wall and the other, applicable in the outer part of the boundary layer, based on the wake model of Clauser (1956). The value of U at the junction of these two representations is denoted by U_m and is determined by conditions that the $\varepsilon/\mu - U$ curve should be continuous with continuous slope at $U = U_m$.

In terms of U and Z the eddy viscosity is represented by:

For $0 \leqslant U < U_m$

$$\frac{\varepsilon}{\mu} = 0.04432[e^{0.4U\sqrt{U_e Z_o Re^{1/2}}} - 1 - 0.4U\sqrt{U_e Z_o Re^{1/2}} - 0.08U^2 U_e Re^{1/2} Z_o] \quad . \tag{26}$$

For $U_m \leqslant U \leqslant 1$

$$\frac{\varepsilon}{\mu} = 0.0168\ U_e Re^{1/2} \int_0^1 (1-U)ZdU \quad . \tag{27}$$

The value of U_m is determined from

$$0.04432[e^{0.4U\sqrt{U_e Z_o Re^{1/2}}} - 1 - 0.4U\sqrt{U_e Z_o Re^{1/2}} = 0.08U^2 U_e Z_o Re^{1/2}$$

$$= 0.0168\ U_e Re^{1/2} \int_0^1 (1-U)ZdU \quad . \tag{28}$$

Yeung and Yang (1981) applied this formulation to three model flows based on data presented at the 1968 Stanford Conference on Turbulent Flows (Coles and Hirst (1968)). The first corresponds to a zero pressure gradient (identified as ID 1400 in the Stanford proceedings), the second to adverse pressure gradient (ID 1100) and the third to favorable pressure gradient (ID 1300). In the first case good results were obtained with the third approximation N = 3 while agreement with experiment was excellent for N = 4, N = 5. The CPU time for N = 4 was only 8 secs (using a CDC 7600 computer). In the third case sufficient accuracy was obtained with N = 3 and higher order approximations were not needed. For adverse pressure gradient flow results were partially satifactory for N = 5 but it is evident that the ε/μ model needs to be improved in this area.

The wall jet problem

Yang and Holt (1983) used the orthonormal version of MIR for turbulent boundary layers to investigate the effect of injecting a parallel stream on turbulent flow in a pipe. This has important applications to coal gasification systems, in which certain products (especially sulfides and oxides) can cause serious corrosion at the walls of pipes in the heat exchange section.

To simplify the problem the flow field is divided into three parts. Firstly, just downstream of the station where the wall jet is introduced, turbulent mixing between the uniform flow in the pipe and uniform flow in the parallel jet is investigated. The injected gas and main stream gas contain different species and both turbulent and molecular diffusion are accounted for. Below the mixing region a turbulent boundary layer develops along the wall of parallel jet and its growth is determined by a separate application of MIR. This second part of the flow field is not initially affected by the mixing process. The third part of the flow field extends downstream of the station where the wall boundary layer and lower part of the mixing layer intersect. The velocity profile in this region of interaction initially has two inflexion points but the orthonormal version of MIR is sufficiently flexible to permit a faithful representation of the Z-U behavior. and the growth of this interacting flow until a full turbulent boundary layer is developed and calculated.

The calculation establishes that introduction of a wall jet can protect a pipe wall from corrosive effects for up to 100 jet thicknesses downstream.

NUMERICAL METHODS FOR UNSTEADY FLOW PROBLEMS

The equations of motion of one dimensional unsteady flow are easily reduced to characteristic form and the earliest numerical methods for solving problems governed by such equations were based on the Method of Characteristics. The latter has the disadvantage of being tied to a non-orthogonal coordinate network which has to be built up, step by step, as the numerical procedure is advanced. To overcome this drawback Godunov (1959) proposed a numerical scheme for solving Lagrangian equations of motion which could be executed in the physical x (distance), t (time) plane. He later extended this scheme to apply to the Eulerian equations in one or more dimensions and used this to solve the Blunt Body Problem. These early Godunov schemes are of first order accuracy and are monotonic in character; that is, if the initial form of the unknown is monotonically increasing this property is preserved as the Godunov schemes are applied at successive time intervals. Godunov (1970) subsequently introduced a scheme of second order accuracy, based on a predictor-corrector approach. This is not monotonic in character but has been successfully applied to many problems in Gas Dynamics with plane symmetry and in Shallow Water Theory. In the period between the publication of the two Godunov schemes, Glimm (1965) proposed a modification of the first Godunov scheme which, in principle, has second order accuracy. We now give a brief comparison between Godunov's first scheme and Glimm's scheme, using explanations of the latter by Chorin (1976,1977). The comparison is made with reference to the one dimensional acoustic equations and is a summary of the discussion in Holt (1984).

Solution of the acoustic wave equation

Consider the one dimensional acoustic wave equation

$$\frac{\partial U}{\partial t} + a\,\frac{\partial U}{\partial x} = 0 \qquad (29)$$

where a is a positive constant (the acoustic speed) with initial conditions

$$U = f(x) \quad , \quad t = 0 \quad . \tag{30}$$

The analytical solution to this equation is

$$U = f(x-at) \quad . \tag{31}$$

If the initial wave form is a Heaviside unit function

$$\begin{aligned} f(x) &= 0 \quad , \quad x \leqslant 0 \\ f(x) &= 1 \quad , \quad x > 0 \end{aligned} \tag{32}$$

the general solution is

$$\begin{aligned} U(x,t) &= 0 \quad , \quad x \leqslant at \\ U(x,t) &= 1 \quad , \quad x > at \end{aligned} \tag{33}$$

representing a step function propagated unchanged along the characteristic line $x = at$.

To solve problem (29), (30) by Godunov's first scheme we divide e x axis into a series of small cells (usually of equal spacing) and represen 'x) by a staircase function so that f(x) is constant in each cell. At e cell boundary there is a discontinuity in the initial value of U and, for t > we solve the problem of breakdown in discontinuity. In other words, we solv a iccession of problems (29), (32). The same procedure is followed at all late. time intervals.

Thus, at a general time (after n time steps each of duration k) we solve the initial value problem: Solve (29) with

$$\begin{aligned} U &= 0 \qquad ih < x \leqslant (i+\tfrac{1}{2})h \quad t = nk \\ U &= 1 \quad (i+\tfrac{1}{2})h < x < (i+1)h \quad t = nk \quad . \end{aligned} \tag{34}$$

If we transfer the origin $\{(i+\frac{1}{2})h,\ nk\}$ to (0,0) this problem has the solution

$$\begin{aligned} U(x,t) &= 0 \quad X \leqslant aT \\ U(x,t) &= 1 \quad X > aT \quad , \end{aligned} \tag{35}$$

where X,T are coordinates. In the Godunov scheme we always use this solution at the cell boundary itself. To satisfy stability of the Godunov scheme we require

$$ah > k \tag{36}$$

so that the boundary $\{(i+1)h,\ (n+\frac{1}{2})k\}$ is to the right of $X - aT = 0$. Therefore,

$$U^{i+1}_{n+\frac{1}{2}} = 1 \quad . \tag{37}$$

so that the Godunov scheme always moves the step jump 0-1 a distance $\frac{1}{2}$ h to the right. As a consequence it can be shown that, after N whole time steps the Godunov scheme causes the step jump to have moved a distance

$$\{h/k - a\}T \tag{38}$$

beyond its position as given by the analytical solution.

In Glimm's scheme we record the solution to the breakdown problem (29),(34) over a half step range on either side of the cell boundary. We then sample this solution at a randomly chosen point within the range (-h/2,h/2) in X. The stability (Courant-Friedrichs-Lewy) condition ensures that the characteristic line $X = aT$ intersects the line $T = \frac{1}{2}k$ inside (-h/2,h/2). The greater flexibility in the Glimm scheme, as compared with the first Godunov scheme, permits us to sample a certain number of intersection points in $(-\frac{1}{2}h,\frac{1}{2}h)$ on both sides of $X = aT$. If the randomly chosen samples are uniformly distributed over (-h/2,h/2) it can be shown that balance between samples to the left and those to the right of $X = aT$ is that required to ensure that the path of the discontinuity 0-1 stays on course. In this sense, then, the Glimm scheme is more accurate than the first Godunov scheme.

Shallow water wave propagation

Li and Holt (1981) applied Glimm's method to the problem of shallow water waves generated by large, near surface, disturbances. The calculation is an extension of the work by Sod (1977) on spherical, or cylindrical, explosions in gases and the paper by Flores and Holt (1981) on underwater explosions. Li and Holt first tested Glimm's method on the classical Dam Break Problem and then applied it to the Dam Break Problem with cylindrical symmetry. It was then used to calculate large amplitude surface waves generated by near surface explosions.

The shallow water equations, omitting dissipative terms, may be written

$$U_t + \{F(U)\}_r = -W(U) \tag{39}$$

where

$$U = \begin{bmatrix} \eta \\ u \end{bmatrix} , \quad F(U) = \begin{bmatrix} u(\eta + d) \\ u^2/2 + g\eta \end{bmatrix} , \quad W(U) = \begin{bmatrix} i(u/r)(\eta + d) \\ 0 \end{bmatrix} \tag{40}$$

and

$$i = 0 \quad \text{for plane symmetry}$$

$$i = 1 \quad \text{for cylindrical symmetry}$$

In Eqs. (40) d is the undisturbed ocean depth, η is the displacement of the ocean surface from its undisturbed position, r is the space coordinate.

Glimm's method is only applicable to equations of motion in conservation form. Following Sod (1977) we therefore solve Eqs. (39) by a splitting method. We solve the homogeneous equations

$$U_t + \{F(U)\}_r = 0 \tag{41}$$

by the established Glimm technique for plane flow equations and subsequently determine the non-homogeneous term from

$$U_t = -W(U) \quad . \tag{42}$$

To apply Glimm's method to Eqs. (41) we use h as a constant space interval and k as the constant time interval. After time $t = nk$ we represent the solution by a staircase function $\tilde{u}_i^n$ such that

$$\begin{aligned} U(r,nk) &= \tilde{u}_{j+1}^n \qquad r > (i + \tfrac{1}{2})h \quad , \\ U(r,nk) &= \tilde{u}_j^n \qquad r < (i + \tfrac{1}{2})h \quad . \end{aligned} \tag{43}$$

Thus U is constant in each cell, with spacing h, along the r axis and jumps discontinuously across each cell boundary. To determine U at $t = (n+\frac{1}{2})k$ we allow each cell boundary to be suddenly removed at $t = nk$ and solve a series of Riemann breakdown problems to determine the solution in $nk \leq t \leq (n+\frac{1}{2})k$. In Glimm's method we sample this solution at a randomly chosen point in the range $(-\frac{1}{2}h, \frac{1}{2}h)$ on either side of a cell boundary. If θ is an equidistributed random variable in $(-\frac{1}{2}, \frac{1}{2})$ Glimm's method then gives

$$\tilde{u}_{i+\frac{1}{2}}^{n+\frac{1}{2}} = U\{((i+\tfrac{1}{2}+\theta)h,\ (n+\tfrac{1}{2})k\} \qquad (44)$$

The grid and the wave fronts used in solving successive Riemann problems are shown in Fig. 1, while the sampling procedure is shown in Fig. 2.

After solving the Riemann problem at $t = (n+\frac{1}{2})k$ the solution (43) is substituted in the right-hand side of Eq. (42) which is solved as a simple difference equation for a corrected $u_{i+\frac{1}{2}}^{n+\frac{1}{2}}$. The corrected values are used as initial data to apply the Glimm method in the next half time interval $(n+\frac{1}{2})k \leq t \leq nk$. The Glimm solution at $t = nk$ is again corrected from Eq. (42).

The solution of the Riemann problem at each cell boundary is given by algebraic formulae, the details of which depend on the nature of the discontinuities across the boundary. Waves are propagated in both directions, when each discontinuity breaks down, and can be either expansion waves or bores (the shallow water equivalent of shocks). The Riemann problem solutions are given in full in Li and Holt (1981).

The initial conditions for the classical Dam Break Problem are shown in Fig. 3 (Fig. 4, Li and Holt). The dam maintains the drop in water level shown. It is suddenly removed at time $t = 0$. Subsequently a bore is propagated to the left while an expansion is propagated to the left. In the Holt-Li calculations a solid wall is introduced to the left of the dam so that reflection of the bore can be calculated. The solutions before and after reflection are shown in Figs. 4 and 4. These calculations were made by the Glimm method alone, since Eq. (42) was not required in the plane flow case. The Holt-Li method was then applied to the cylindrical dam break problem. In this case a cylindrical bore converged on its center and was reflected there, in a manner similar to a cylindrical implosion in a gas. The wave profiles in this case are shown in Fig. 6.

The final calculation deals with the Upper Critical Depth problem. This concerns the generation of large amplitude surface waves by spherical explosions detonated at different depths below, but near to, the ocean surface. The pressure field from the underwater explosion was calculated previously by Falade and Holt (1978). The data were used to provide initial conditions in the surface wave calculation using Glimm's method. Figure 7 shows the most important result of this calculation namely, that for different depths of explosive charge the maximum amplitude of surface wave generated occurs when the charge depth is equal to one half of the charge radius. This confirms the Upper Critical Depth phenomenon observed in field tests.

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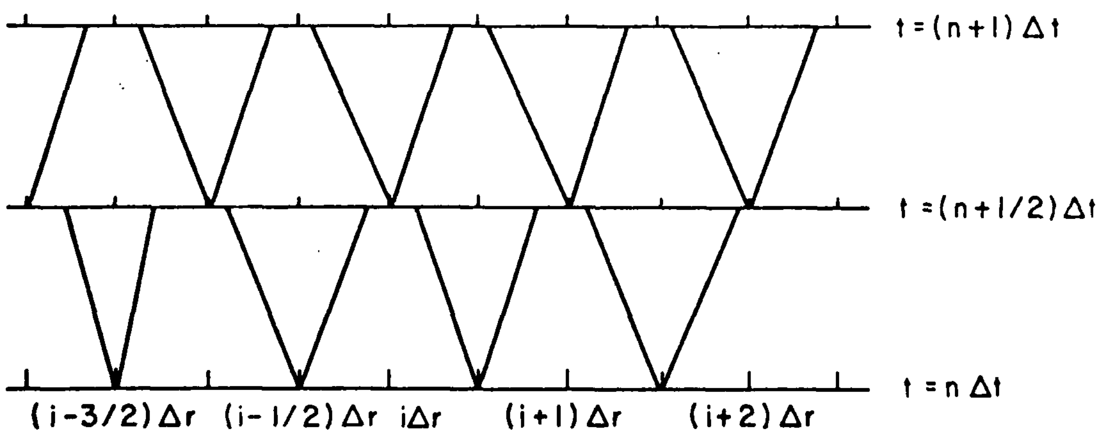

Fig. 1. Sequence of Riemann problems on grid.

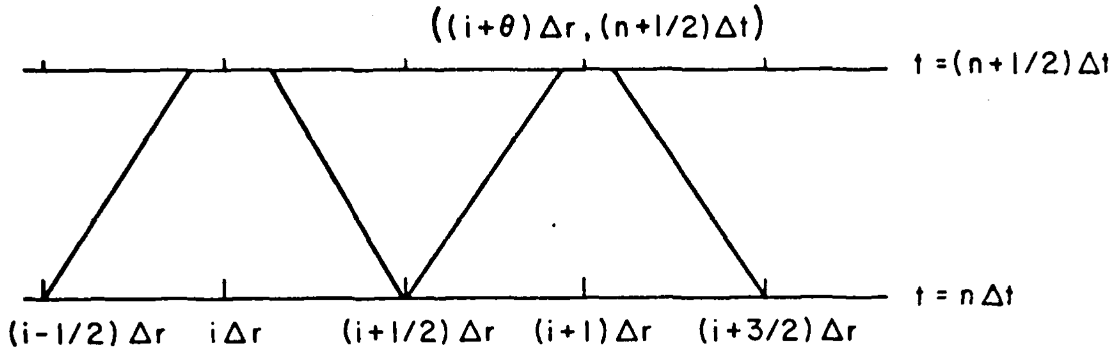

Fig. 2. Sampling procedure for Glimm's scheme.

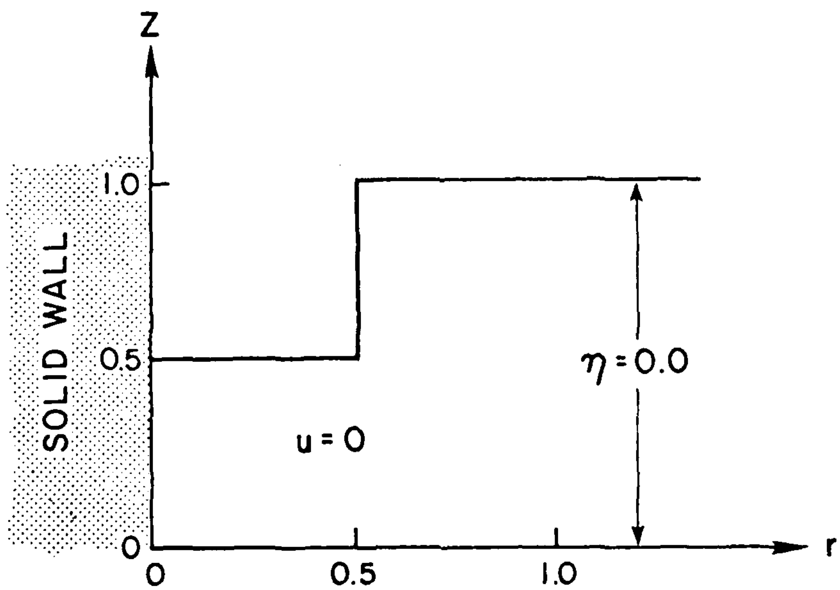

Fig. 3. Initial condition for dam break problem.

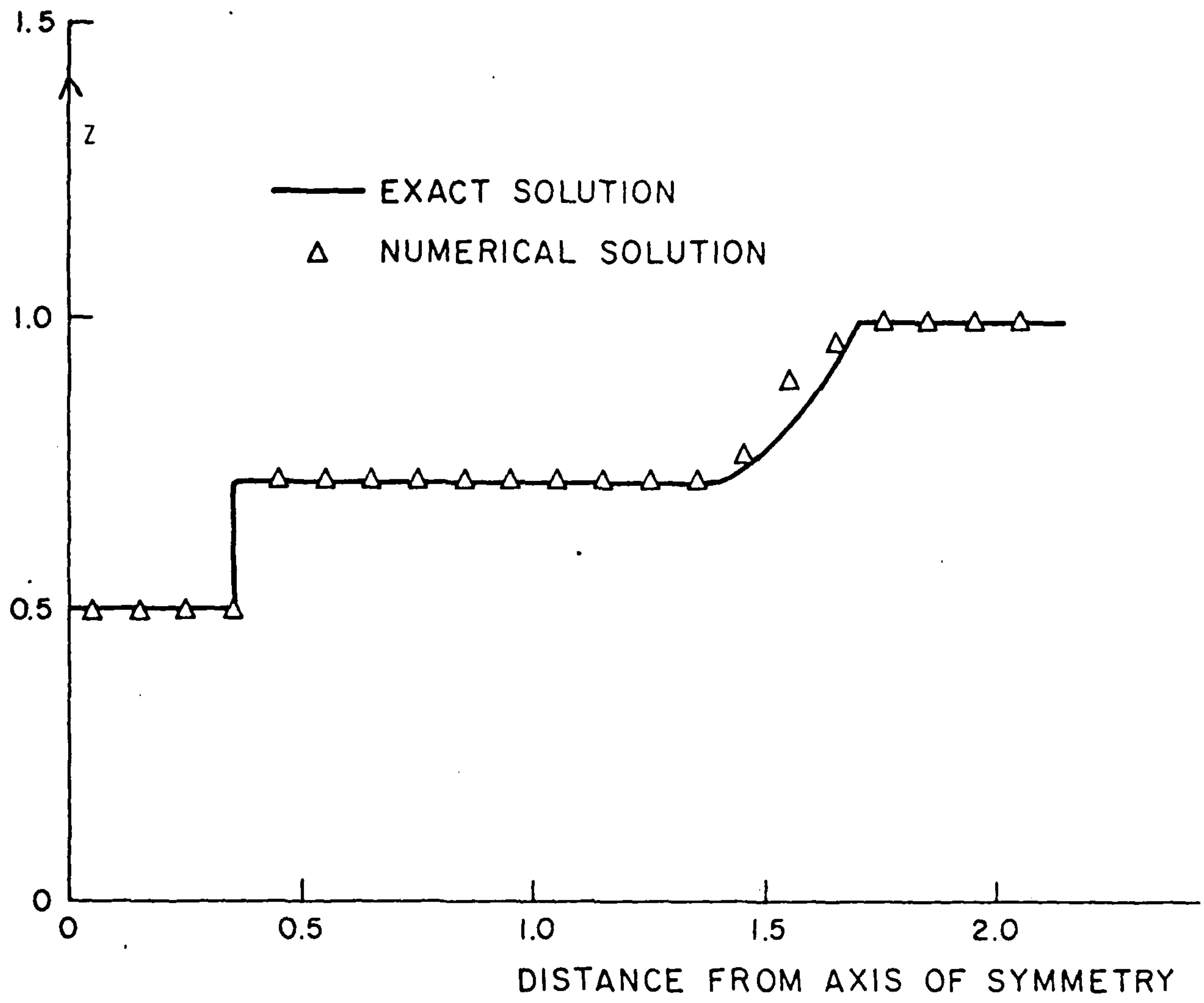

Fig. 4. Dam break problem with plane symmetry.
Time t =0.69.

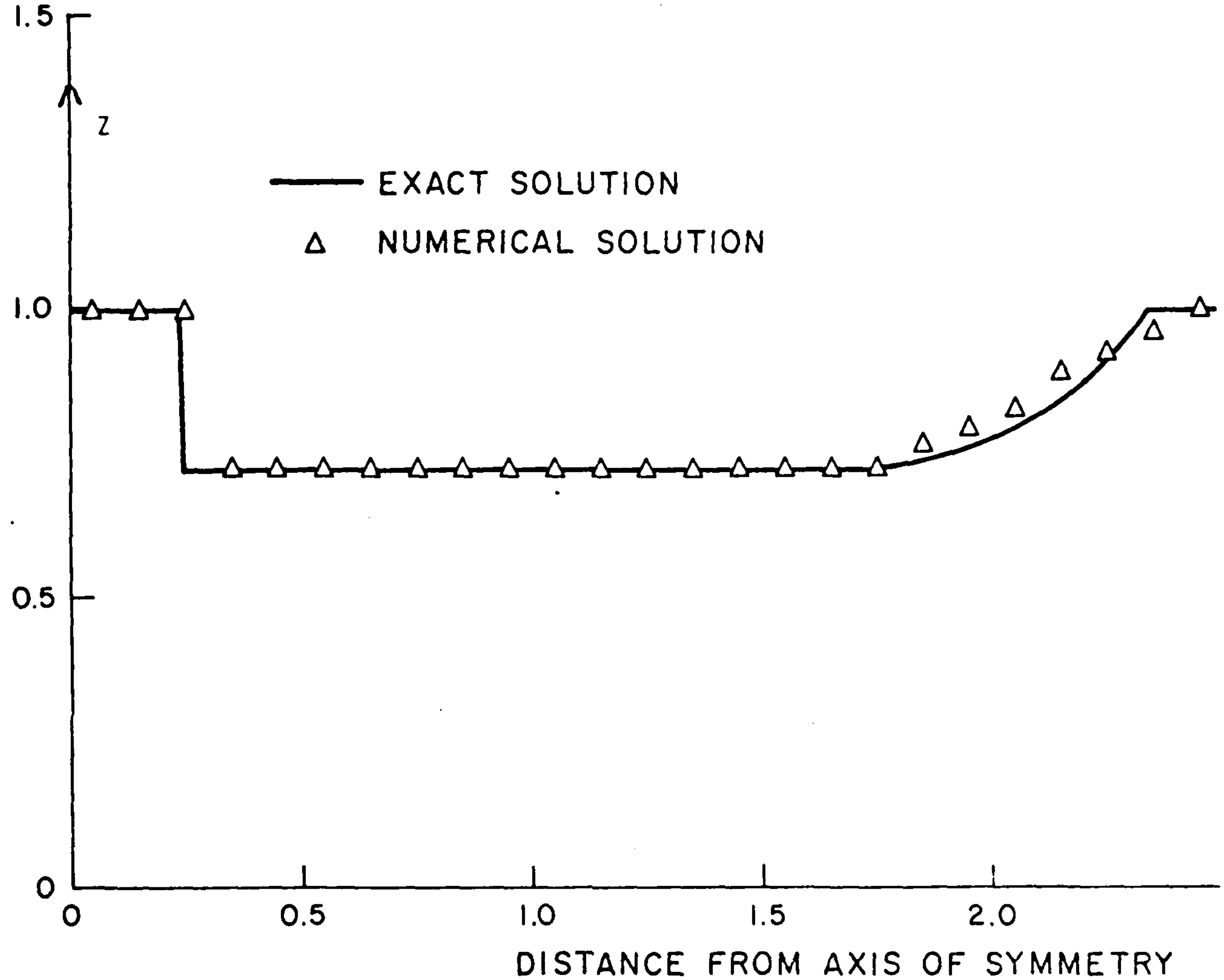

Fig. 5. Dam break problem with plane symmetry.
Time t = 1.36.

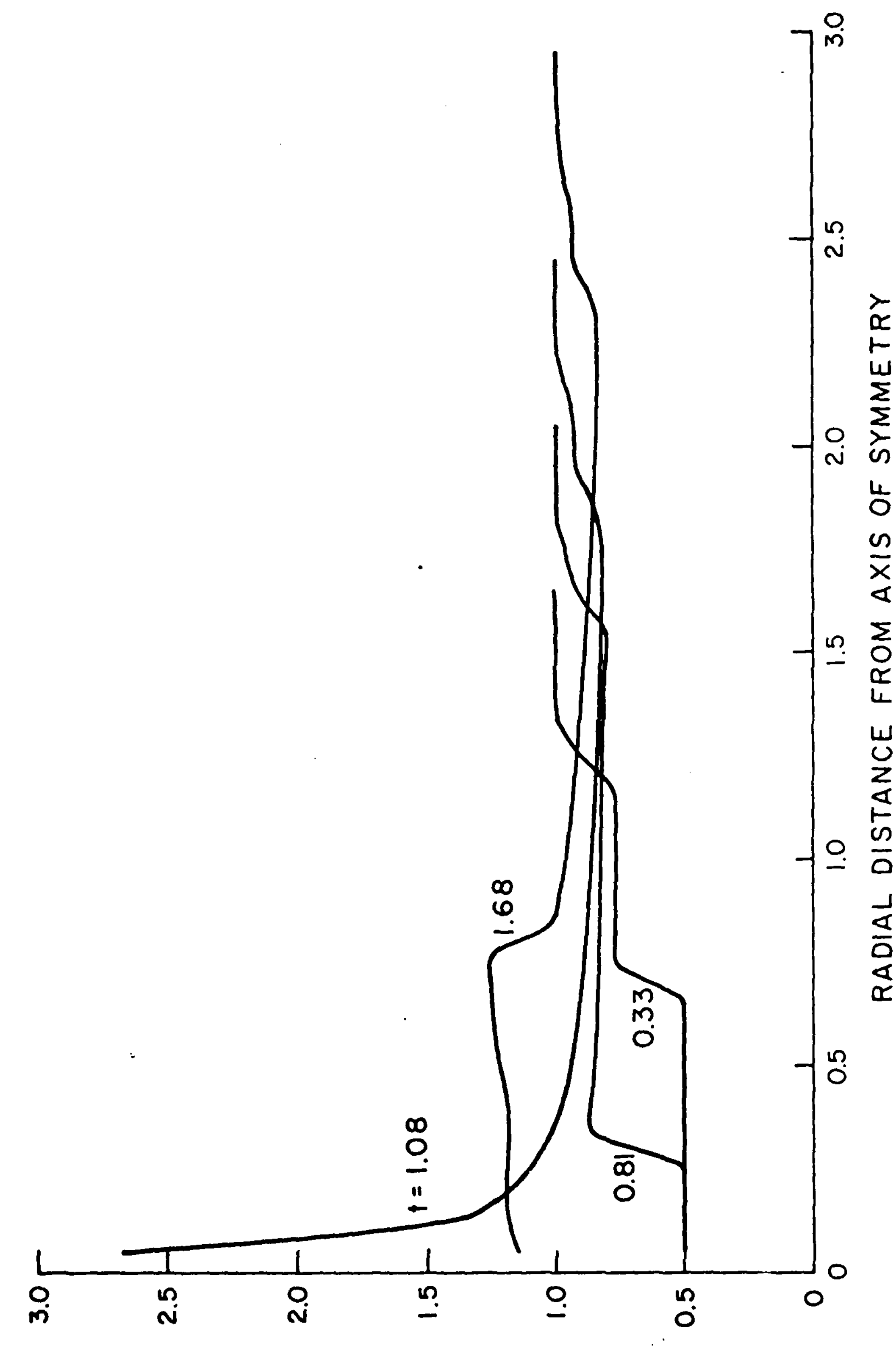

Fig. 6. Dam break problem with cylindrical symmetry.

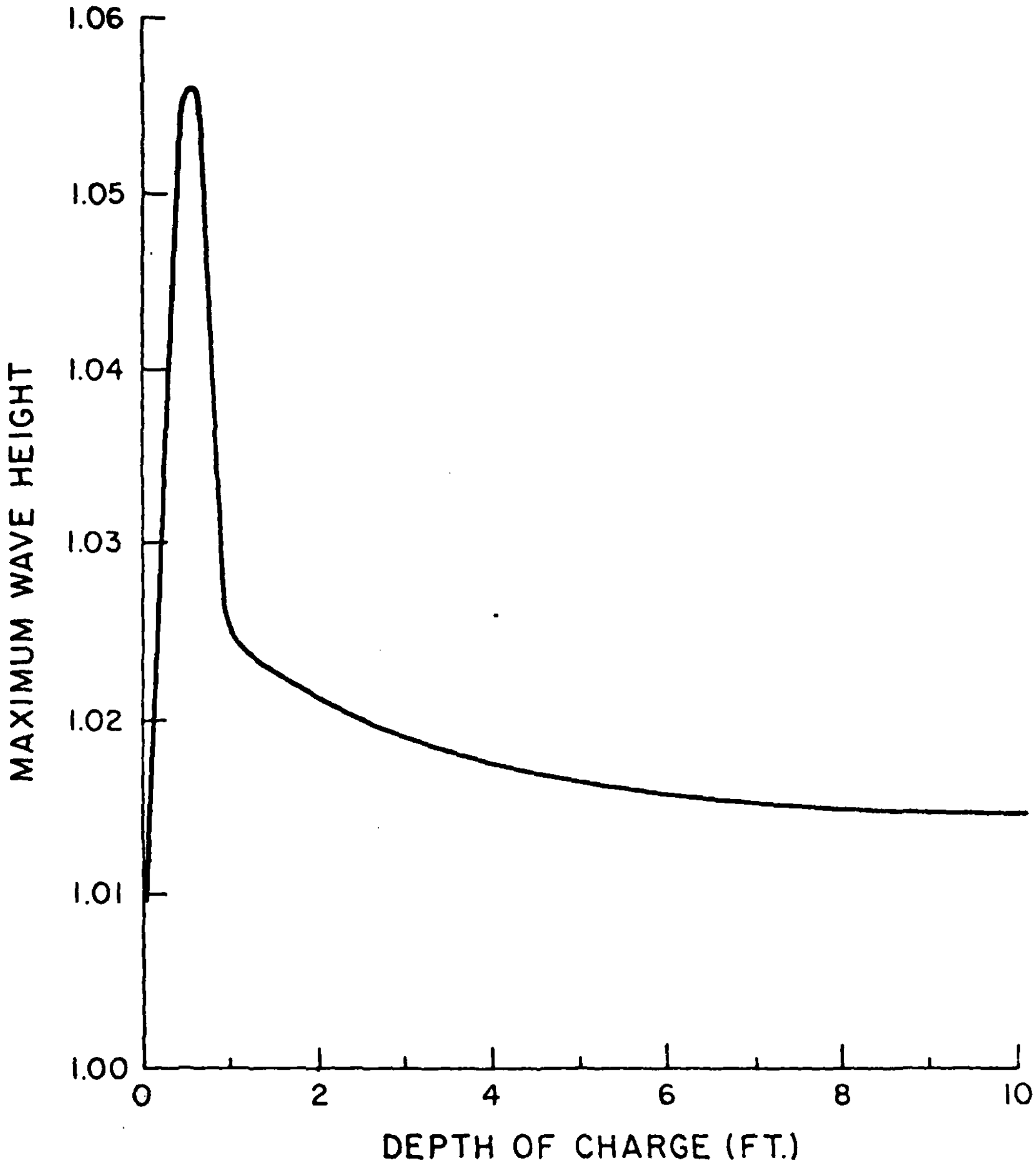

Fig. 7. Variation of maximum wave height with charge depth.